

50. We orient our $+x$ axis along the initial direction of motion, and specify angles in the “standard” way – so $\theta = -90^\circ$ for the particle B which is assumed to scatter “downward” and $\phi > 0$ for particle A which presumably goes into the first quadrant. We apply the conservation of linear momentum to the x and y axes respectively.

$$\begin{aligned} m_B v_B &= m_B v'_B \cos \theta + m_A v'_A \cos \phi \\ 0 &= m_B v'_B \sin \theta + m_A v'_A \sin \phi \end{aligned}$$

(a) Setting $v_B = v$ and $v'_B = v/2$, the y -momentum equation yields

$$m_A v'_A \sin \phi = m_B \frac{v}{2}$$

and the x -momentum equation yields

$$m_A v'_A \cos \phi = m_B v .$$

Dividing these two equations, we find $\tan \phi = \frac{1}{2}$ which yields $\phi = 27^\circ$. If we choose to measure this from the final direction of motion for B , then this becomes $90^\circ + 27^\circ = 117^\circ$.

(b) We can *formally* solve for v'_A (using the y -momentum equation and the fact that $\sin \phi = 1/\sqrt{5}$)

$$v'_A = \frac{\sqrt{5}}{2} \frac{m_B}{m_A} v$$

but lacking numerical values for v and the mass ratio, we cannot fully determine the final speed of A . Note: substituting $\cos \phi = 2/\sqrt{5}$, into the x -momentum equation leads to exactly this same relation (that is, no new information is obtained which might help us determine an answer).